SURFACE RECONSTRUCTION FROM MULTI-VALUED GRID DATA

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Abstract The paper presents a graph based combinatorial approach for reconstruction of multi-valued surfaces from the data given in the form of a grid. The input is assumed to be given in the form of a set of quintuplets (i,j,x,y,z) where (i,j) are the grid indices and (x,y,z) are the coordinates of the corresponding point. The given points are first grouped into the sets with identical i-values and sets with identical j-values. For each such set then a novel graph-based curve-fitting algorithm is employed which can generate curves with branch points and discontinuities. These sets of curves are then combinatorially matched to obtain networks of loops, which on tessellation gives the reconstructed surface. The illustrative examples demonstrate that even for apparently simple data set, a topologically wide variety of surfaces can be generated. The present method can handle single valued, multi-valued, manifold, non-manifold surfaces with or without holes.

Keywords: Reconstruction, grid, surface-fitting, non-manifold.

INTRODUCTION

Surface reconstruction refers to the variety of procedures used to obtain a surface representation of objects from which data has been obtained as coordinates of points that lie on the bounding surface of a real object. Thus surface reconstruction algorithms assumes the presence of an underlying real surface and aims at modeling it from measured data. The source of data can be quite diverse such as digital or ultrasonic scanning of engineering or biological objects, seismic survey of subterranean geological features, range imaging, data obtained from scanning tunneling microscopes (for extremely small objects), data synthetically generated from image processing of X-ray or angiographs, CT scans etc. A reconstruction algorithm generally depends on the structure of the data. The accuracy and topology of the resultant surface depends on the method employed for fitting the data. Smooth fitting of data is mostly inexact and gives a four-cornered patch, which has severe topological limitations. Tessellated surfaces can represent objects with arbitrary topology and sharp local features and can fit the data exactly. In the present work, a grid structure of the data is assumed which is typically obtained from post processing of seismic survey of subterranean geological features and a tessellated surface fitting is performed.

Reconstruction of single valued surfaces from grid data is quite straightforward [Verhoff, *et al.*, 1989]. The authors of this article are not aware of any work available in literature for surface fitting on multi-valued grid. Substantial amount of work is, however, available that fits tessellated or smooth surface over a dense cloud of points (some of which use grid as an intermediate data structure, they are not meant to handle gridded data). Tessellated surface fitting algorithms are mostly computational geometry based that work for well-distributed points and do not utilize the possible structure in the data. [Hoppe *et al.*, 1992] estimates the tangent plane at each sample point using *k*-nearest neighbours and then uses marching cube algorithm to actually obtain the tessellated surface. [Edelsbrunner and Mucke, 1994] extended the concept of α-shapes to 3D which basically eliminates potentially interior points and produces a triangulation for the remaining points. The topology of the result is unpredictable and sensitive to the parameter α, which is difficult to estimate. [Attali, 1998] used the concept of γ-regular shapes in mathematical morphology to reconstruct surface from unorganized points. More recent works assume that point cloud belong to the boundary surface of the solid, possibly generated by scanning or use of a coordinate measuring machine. [Amenta and Bern, 1999] proposed voronoi filtering [Amenta *et al.*, 2000] used complimentary cone for reconstruction of complicated surfaces from sufficiently dense and uniform point cloud. Other methodologies relevant for tessellated surface fitting over point cloud can be found in [Ruud. and Vemuri, 1991, Bernardini *et al.*, 1999, Pulli and Shapiro, 2000, Lee, 2000, Sun *et al*, 2001, Floater and Reimers, 2001]. Reconstruction of smooth surfaces from point cloud is generally an approximate one and mostly requires segmentation of the data based on computed differential property of the underlying surface [Loop and DeRose, 1990; Sarkar and Menq, 1991, Sapidis and Besl, 1995] .

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DEFINITIONS

Grid: A grid is a set of points whose projection on *xy*-plane can be arranged such that the points are located at the intersections of two orthogonal sets of parallel equispaced lines. Thus, every point can be associated with two integers, called indices, identifying the particular pair of lines at the intersection of which its projection is located. A point in a grid can, thus, be represented by as (i,j,x,y,z) where (i,j) and (x,y,z) correspond to the grid indices and coordinates of the point.

Multi-valued Grid: A grid data set in which at least one (i,j) value occur more than once is called a multi-valued grid.

Layer: The set of data points with a given value of *i* or *j* lies on a plane called a layer.

Level: The set of data points in a layer with a given value of *j* or *i* lies on a line called a level.

Edge: A line segment joining two points in two consecutive levels is called an edge.

Loop: A sequence of edges forming a closed simple polygon in 3D space is called a loop.

Slope: The angle made by an edge with *z*-plane is called the slope or absolute slope of the edge. Angle made by an edge with its adjacent edge is called the relative slope of the edge.

ASSUMPTIONS

1. Data is noiseless
2. Data is available in the form of a grid, *i.e.*, as a set of (i,j,x,y,z) values.
3. Data is complete, *i.e.*, for a given (i,j) value all the (x,y,z) values on the underlying surface are given.
4. Holes on the surface manifest as absence of grid points in the data within the region of interest.
5. Density of data is such that a piecewise linear interpolation through the data is adequate.
6.

OVERVIEW

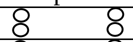
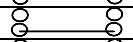
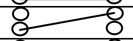
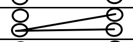
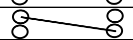
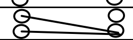
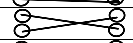
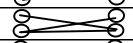
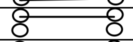
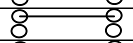
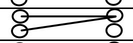
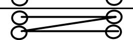
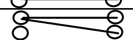
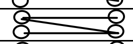
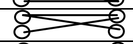
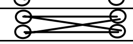
The objective of the present approach is to obtain all possible solutions satisfying certain conditions. Here we follow a bottom-up approach; *i.e.* first, for the points in each layer, the 2D curve-fitting♣ problem is solved by a graph-based algorithm, then the curves in the orthogonal planes are matched to form four-sided loops which are then tessellated by adding a diagonal to obtain two triangles per loop. The assembly of all triangles gives the required surface. Since in a multi-valued grid there are many ways the points can be joined to form the set edges in a layer and there are many ways to form the loops, it is possible to generate a large number of solution. Hence, to capture more practical solutions, constraints are imposed to restrict undesirable fragmentation of the resultant surface and to avoid undue variation of slope.

CURVE FITTING

Data read as grid is first organized in the form of layer-level hierarchy in both *x* and *y* directions. For each layer the curve-fitting problem is solved by first constructing the bipartite graphs between each pair of adjacent levels and then concatenating the valid graphs to obtain the curves in a layer.

Bipartite Graph: The levels in a *j*-layer are arranged in an ascending order of their *i*-values and the points in a level are sorted in the ascending order of their *z*-values. Now all possible bipartite graphs are considered between the points in the leftmost pair of levels and each such graph is geometrically validated; if the edges in the graph intersect or the slope of any edge is more than a prescribed value, the graph is considered invalid and discarded. Since the points are already sorted, intersection check can be done symbolically which is fast, robust and reliable. Two edges e_1 and e_2 respectively in a layer between points p_1, p_2 and q_1, q_2 do not intersect if the *z*-coordinates of p_1-q_1 and p_2-q_2 have same sign. From table-1 it can be observed that for a case of two points per level, although the possible number of bipartite graph 16, only 4 of them pass both the tests with allowed absolute slope to be 0°.

Table 1: Generating valid bipartite graphs

option	Graph	intersection	slope
1		Passed	Passed
2		Passed	Passed
3		Passed	
4		Passed	
5		Passed	
6		Passed	
7			
8			
9		Passed	Passed
10		Passed	Passed
11		Passed	
12		Passed	
13		Passed	
14		Passed	
15			
16			

Concatenation: It is the step in which bipartite graphs are concatenated to obtain the feasible set of curves on each layer. Each pair of consecutive levels in a layer generates a number of valid graphs. If n_i is the number of valid graphs in the levels l_i and l_{i+1} in a layer, and there are $k+1$ such levels, then the total number of ways for concatenating the graphs to obtain the curves in the layer is $n_1 \times n_2 \times \ldots \times n_k$. It can be observed that trivial bipartite graphs are allowed which on one hand allows variety of topologies to be created but on the other hand

♣piecewise linear curve with possible branch points and discontinuities fragments the curve. To keep the total number of solutions to a manageable value and to eliminate generation of invalid surfaces in subsequent steps following conditions are imposed.

1. All the points in the shared level must have at least one edge incident on them
2. The angle between an edge in one graph adjacent to an edge in the adjacent graph (relative slope) must be within a given value.
3. The minimum number of levels spanned by each connected component (fragment length) must be more than some prescribed value.

Fig.1 illustrates the effect of these constraints in validation of results of curve construction in a layer.

SURFACE FITTING

Generation of reconstructed candidate surfaces from the two sets of curves belonging to the *i*-layers and *j*-layers has two steps: loop generation and loop tessellation.

Loop generation: For a given set of curves on the *i*-layers and *j*-layers, the algorithm to construct a loop associated with the grid point (*i*,*j*) is given below.

1. On layer *j*, select an edge between levels *i* and *i+1*. If no such edge is found, discard the option and try with the next set of curves and restart from step-1.
2. Switch to layer *i+1* and select an edge between levels *j* and *j+1* such that it shares a vertex with the edge selected in step-1. If no such edge is found, discard the option and try with the next set of curves and restart from step-1.
3. Switch to layer *j+1* and select an edge between levels *i+1* and *i* such that it shares a vertex with the edge selected in step-2. If no such edge is found, discard the option and try with the next set of curves and restart from step-1.
4. Switch to layer *i* and select an edge between levels *j+1* and *j* such that it shares one vertex with the edge selected in step-1 and the other vertex with the edge selected in step-3. If no such edge is found, discard the option and try with the next set of curves and restart from step-1.
5. Store the four edges so detected in proper order to form a loop.
6. Follow step-1 to step-5 to identify all possible distinct loops associated with grid point (*i*,*j*).

Loop tessellation: If *a, b, c, d* are the four vertices in a four-sided loop the triangles *abc* and *acd* gives a tessellation of the loop.

The above steps are followed for each value of *i* and *j* of interest to obtain an exhaustive set of loops for a given set of curves on the respective layers. The set of all such triangles obtained from all the loops detected in the above step gives a reconstructed surface. Same steps are followed for all the sets of curves on the layer to generate the set of all possible reconstructed surfaces.

RESULTS AND DISCUSSION

The above algorithm has been implemented using C++ and OpenGL has been used for visualization. Several real and synthetic data was used to demonstrate the capabilities of the proposed method. It has been established that the algorithm can generate all types of topology of surfaces: manifold, non-manifold, open, closed and surface with holes. However, for the sake of brevity only a representative set of results is included in this article. Since the main purpose of the work was to study the topological variety of the surfaces that get generated and its sensitivity to the user specified parameters, not much effort has been put to optimize the algorithm and its implementation. The time taken depends on number of data points, absolute and relative slope and minimum fragment length specifications in a non-linear fashion. This is because the number of solutions grows very fast with reduced fragment length and relaxed slope constraint and hence it takes more time.

In the figures, the four numbers in the bracket correspond to absolute slope, relative slope, minimum number of points in a curve and number of valid surfaces generated (from which only a representative set is included in the figure).

Table-2 shows the statistics for one set of control parameters, which demonstrates how the number of solutions is brought to a manageable value. A representative set of corresponding results is shown in Fig.2.

Table 2 Effect of control parameters on reconstruction from data in Fig.2

Algorithm step	Statistics
Number of data points	84
Layers in *i*-direction	8
Layers in *j*-direction	3
Absolute slope	45^o
Relative slope	45^o
Curves generated in a layer (average)	29594
Curves without isolated points (average)	4901
Curves with at least 5 points (average)	8
Surfaces from valid curves	512
Valid surfaces generated	17

Using a small set of regular array of points a wide variety of surface topologies are generated by the above method (Fig.3). The results can be useful in study of crystallographic planes. The time taken for generating the results is sensitive to the control parameters. For the different combinations of parameters tried the time varied from 3 seconds to 30 minutes. If the data is a single valued grid, the present methodology very fast and generates a unique surface (Fig.4). On the other hand, missing data points are likely to manifest in the form of holes in the surface (Fig.5). If the data is a of more general nature, many interesting combination of closed, and open surfaces might be generated which are basically non-manifold in nature (Fig.6) and difficult to obtain by the existing methods.

CONCLUSIONS

The paper presented a simple graph based combinatorial approach to the problem of surface reconstruction from multi-valued grid. The method fully exploits the inherent structure in the data and generates all possible tessellated surfaces satisfying the slope and fragment length constraints. These constraints are imposed to avoid exorbitantly high number of (combinatorially possible) solutions for a non-trivial data set, only some of which are practically meaningful. It is demonstrated that the proposed method can generate all variety of surfaces: manifold, non-manifold, open, closed and those with holes.

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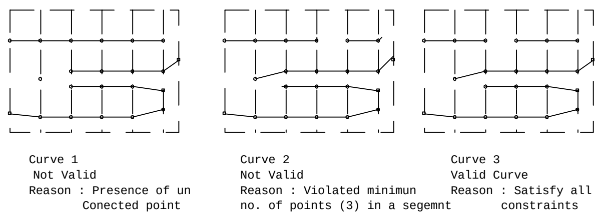

Fig.1 Validation of a curve

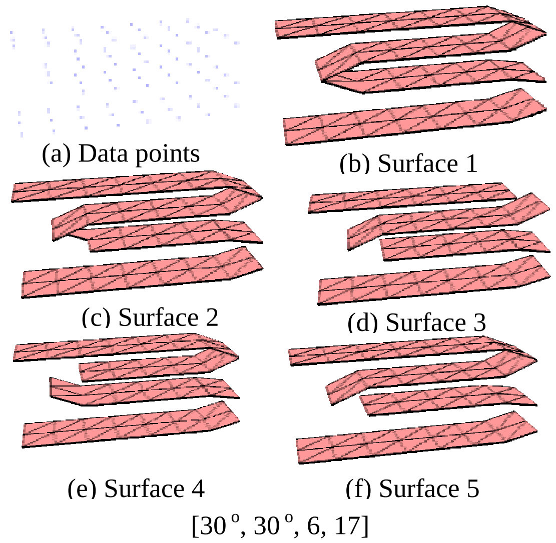

Fig.2 Reconstructing a multi-valued surface

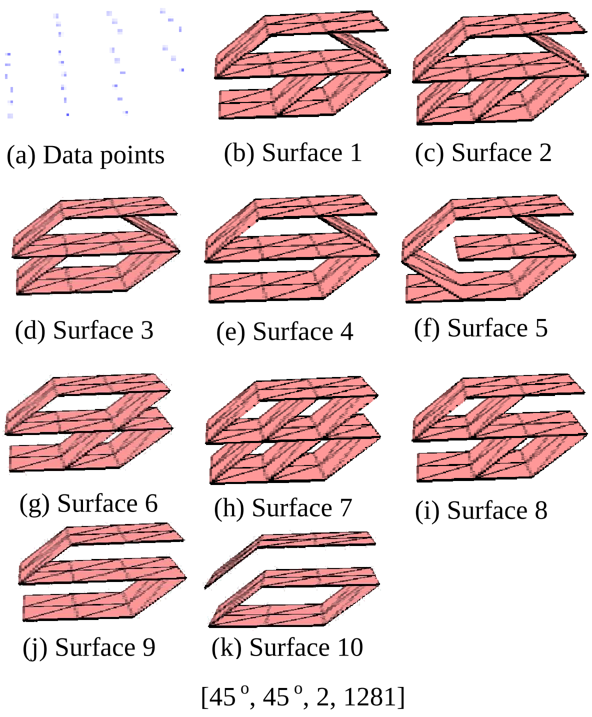

Fig.3 Multiple results from a simple data set

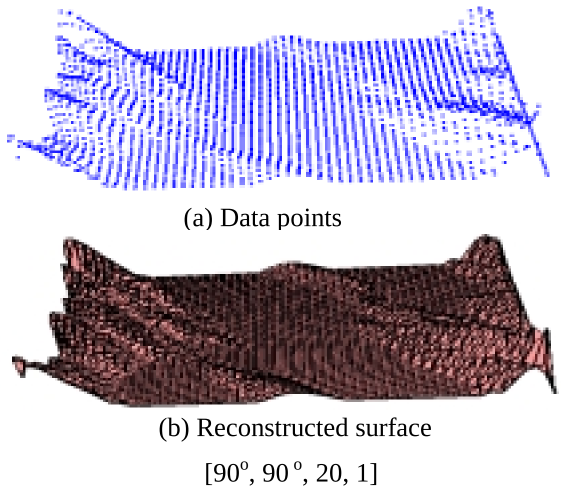

Fig.4 Reconstruction of single valued surface

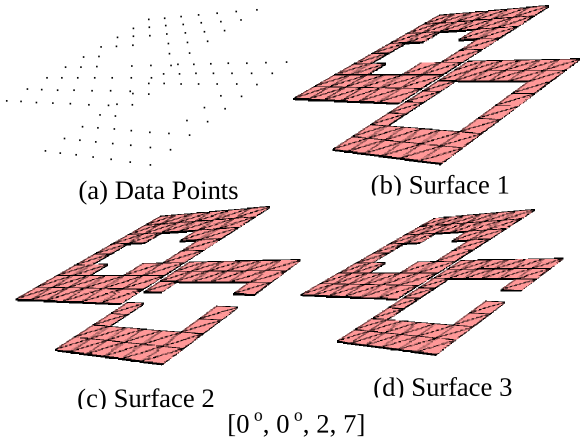

Fig.5 Surface with hole

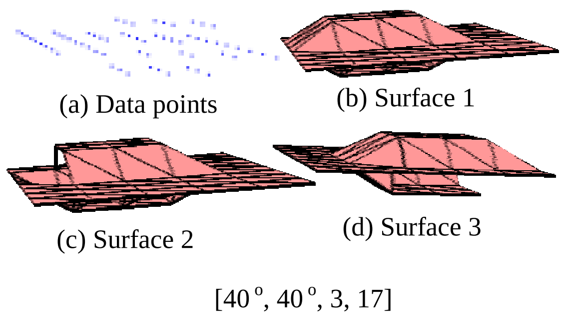

Fig.6 Non-manifold surface with closed and open portions